AD-A187 130   ANNUAL REPORT ON RESEARCH SPONSORED BY GRANT   1/1
              AFOSR-84-0159(U) CALIFORNIA UNIV DAVIS INTERCOLLEGE DIV
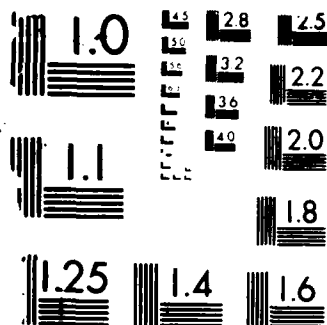   OF STATISTICS F J SAMANIEGO 31 JUL 87 AFOSR-TR-87-1461
UNCLASSIFIED AFOSR-84-0159   F/G 12/3   NL

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During this period, research has been completed in a number of distinct areas. The results obtained, as well as the goals of a number of related ongoing investigations, are detailed below:

1. <u>Estimation in the NBU-{t_0} Class of Survival Distributions</u> (Reneau, Samaniego)

Suppose the survival function S of a random variable X is a member of the NBU-{t_o} class. By definition, S must satisfy the following condition:

(1) $$S(x+t_0) \leq S(x) \, S(t_0) \quad \text{for all } x \geq 0.$$

Let {$X_1, \ldots, X_n$} be a random sample from S. Define the indicator function of the set A to be

$$I_A(x) = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{otherwise} \end{cases}$$

S may be estimated using the empirical survival function S_n, defined by

$$S_n(t) = \frac{1}{n} \sum_{i=1}^{n} I_{(t,\infty)}(X_i) \, .$$

$S_n(t)$ is simply the proportion of observations in the sample which are greater than t. S_n is a strongly consistent estimator of S which converges to S at an optimal rate. However, S_n may not necessarily satisfy condition (1), i.e., S_n need not be an NBU-{t_o} distribution. Our

REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION		1b. RESTRICTIVE MARKINGS		
UNCLASSIFIED				
2a. SECURITY CLASSIFICATION AUTHORITY		3. DISTRIBUTION/AVAILABILITY OF REPORT		
N/A		Approved for public release;		
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE		distribution unlimited		
N/A				
4. PERFORMING ORGANIZATION REPORT NUMBER(S)		5. MONITORING ORGANIZATION REPORT NUMBER(S)		
		AFOSR-TR- 87-1461		
6a. NAME OF PERFORMING ORGANIZATION	6b. OFFICE SYMBOL (If applicable)	7a. NAME OF MONITORING ORGANIZATION		
University of California Davis, CA 95616		AFOSR/NM		
6c. ADDRESS (City, State and ZIP Code)		7b. ADDRESS (City, State and ZIP Code)		
Division of Statistics University of California Davis, CA 95616		Bldg. 410 Bo-ling AFB, DC 20332-6448		
8a. NAME OF FUNDING/SPONSORING ORGANIZATION	8b. OFFICE SYMBOL (If applicable)	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER		
AFOSR	NM	AFOSR 84-0159		
8c. ADDRESS (City, State and ZIP Code)		10. SOURCE OF FUNDING NOS.		
Bldg 410 Bolling AFB, DC 20332-6448		PROGRAM ELEMENT NO.	PROJECT NO.	TASK NO.	WORK UNIT NO.

11. TITLE (Include Security Classification)	PROGRAM ELEMENT NO.	PROJECT NO.	TASK NO.	WORK UNIT NO.
Annual Report on AFOSR 84-0159, July 1, 1986 through June 30, 1987	6.1102F	2304	A6	

12. PERSONAL AUTHOR(S)
F.J. Samaniego, Principal Investigator

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Yr., Mo., Day)	15. PAGE COUNT
Interim	FROM 7/1/86 TO 6/30/87	July 31, 1987	9

16. SUPPLEMENTARY NOTATION

17	COSATI CODES		18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB. GR.	

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

Research accomplished during the period July 1, 1986 - June 30, 1987 supported in part under grant AFOSR 84-0159 is described.

20. DISTRIBUTION/AVAILABILITY OF ABSTRACT	21 ABSTRACT SECURITY CLASSIFICATION	
UNCLASSIFIED/UNLIMITED ☒ SAME AS RPT ☐ DTIC USERS ☐	UNCLASSIFIED	
22a. NAME OF RESPONSIBLE INDIVIDUAL	22b TELEPHONE NUMBER (Include Area Code)	22c OFFICE SYMBOL
Brian W. Woodruff		AFOSR/NM

goal is to find an estimator that itself belongs to the NBU-$\{t_0\}$ class and has the same asymptotic optimality properties as S_n.

In this paper, the estimator $\hat{S}$ of S, given implicitly by

$$(2) \qquad \hat{S}(x) = \begin{cases} S_n(x) & 0 \leq x \leq t_0 \\ \min\,[S_n(x),\ \hat{S}(x-t_0)\hat{S}(t_0)] & x > t_0 \end{cases} \ ,$$

is studied. The following results are obtained:

Theorem 1: Condition (2) uniquely defines an NBU-$\{t_0\}$ survival function which may be written as a function of S_n as follows:

$$(3) \qquad \hat{S}(x) = \min_{0 \leq k \leq [x/t_0]}\left\{\, S_n^k(t_0)\ S_n(x-kt_0)\,\right\}\ ,$$

where $[u]$ denotes the greatest integer less than or equal to u.

Theorem 2: $\hat{S}$ is the largest NBU-$\{t_0\}$ survival function which is less than or equal to S_n for all $x \geq 0$.

Theorem 3: If S is in the NBU-$\{t_0\}$ class, then $\hat{S}$ converges uniformly to S with probability 1.

Theorem 4: The rates of mean square and almost sure convergence of $\hat{S}$ to S are given by:

$$E|\hat{S}(x)-S(x)|^2 = O(1/\sqrt{n})$$

and

$$|\hat{S}(x)-S(x)| = O\left(\left[\frac{\log\,\log\,n}{n}\right]^{1/2}\right)$$

These results establish the fact that the estimator $\hat{S}$, given implicitly in (2) and explicitly in (3) above, is a strong uniformly consistent NBU-$\{t_0\}$ estimator of S which converges to S at the best possible (pointwise

and mean square) rates. Confidence procedures based on $\hat{S}$ and also discussed. A technical report on this work will appear in Summer, 1987.

2. Estimation of Survival Distributions which are New Better Than Used With Respect to a Distinguished Set. (Reneau, Samaniego)

For any set $A \subseteq [0,\infty)$, we define a survival function S to be New Better Than Used with respect to the Set A (NBU-A) if

$$S(x|t) \leq S(x|0) \quad \text{for all} \quad x \geq 0, \ t \in A ,$$

where $S(x|t) = P(X > x+t | x > t)$. The condition is equivalent to

(4) $$S(x+t) \leq S(x) \, S(t) \quad \text{for all} \quad x \geq 0, \ t \in A .$$

This class clearly contains the NBU class of distributions, and it is in turn contained in the NBU-{t} class for all $t \in A$. In fact, we can write

(5) $$\text{NBU-A} = \bigcap_{t \in A} \text{NBU-}\{t\} .$$

Recursive estimation formulas such as (2) above are studied for a number of special cases. When A is a finite set, the resulting estimators are shown to be strong uniformly consistent estimators of the corresponding survival curves. The same is true for the set $A = \{k \, t_o : k \, J \, 1\}$. When the set A contains a limit point, the methods used on the preceeding cases are not, in general, successful in producing a consistent estimator. They do provide useful estimators in the following important application.

Consider the class of survival functions S in the NBU-A class for which there exists $\delta > 0$ such that $S(x) = 1$ for $0 \leq x \leq \delta$. One situation where this type of survival function occurs in practice is when products

are subjected to burn-in. It is common for manufactured items to experience a disproportionate number of failures during the early portion of their lifetimes. Dhillon and Reiche (1985) list inadequate quality control and manufacturing methods, substandard materials and workmanship, wrong start-up and installation, difficulties in assembly, inadequate handling methods and wrong packaging as some of the reasons for these early failures. One method to deal with this phenomenon is the burn-in process in which items are subjected to a preliminary screening, and those that fail before a predetermined time δ are discarded. The survival distribution S of the items which survive the burn-in period will satisfy the condition $S(x) = 1$ for $x \leq \delta$.

The survival function obtained by applying left-truncation to a survival function S,

$$S_\delta(x) = \begin{cases} 1 & \text{if } 0 \leq x < \delta \\ S(x) \; / \; S(\delta) & \text{if } \delta \leq x \end{cases}$$

is itself in the NBU-A class whenever S is. Thus, random sampling from a NBU-A distribution under fixed left-truncation such as burn- in yields a survival function of the desired type. When S has the property that $S(\delta) = 1$ for some $\delta > 0$, the recursive approach to estimating S is shown to lead to a strong uniformly consistent estimator with optimal convergence rates. A technical report on this work is in preparation.

3. On Comparing Coherent Systems. (Samaniego)

Various methods and criteria for comparing coherent systems are discussed. A theoretical result is derived for comparing systems of a given order when components are assumed to have independent and identically distributed lifetimes. The result gives sufficient

conditions for the lifetime of one system to be stochastically larger than that of another system. This paper has been submitted for publication.

4. On Failure Rate Estimation When Test and Operating Environments Differ. (Samaniego)

Lindley and Singpurwalla [2] have studied exponentially distributed failure time data when components are tested in two different environments and each component failure rate in the second environment is a constant multiple of its failure rate in the first environment. In this paper, the maximum likelihood estimator of this model's parameter vector is derived in terms of the unique positive root of a specific rational function. The estimator of component failure rates is compared to the MLE of these failure rates when no scale-change assumption is made. This paper has been submitted for publication.

5. Joint Life-Time Distributions. (Ghurye)

The class of distributions having the lack-of-memory property was extended to incorporate simple aging characteristics. These results have now been published in Advances in Applied Probability (April 1987).

Since such a distribution depends on a large number of parameters, considerable simplification is achieved by restricting attention to a system all of whose components are identical; in such a situation, the life-times are exchangeable variables, which reduces the analytical complexity to the minimum possible (with the exception of the trivial case of mutually independent components). We are then dealing with a system (such as "k-out-of-n" which has several operational states, and can be maintained in operational condition by occasional replacement of

some of the failed components. We have now shown that the solution of this latter problem depends only on (more-or-less) routine computations.

6. Stochatic Differential Equations for Repair Policies. (Ghurye)

Optimization problems of multi-component systems involve large, though finite, state-spaces (as in the example discussed in the previous section); such problems are usually soluble by specific computations in each case, but not in terms of general formulae which help in the understanding of the basic phenomena. Therefore, it might be worthwhile to replace the discrete sets by continuous idealizations; this approach has proved fruitful. A brief description follows.

Let X(t) denote the state of the system at time t (the state being measured by the level of deterioration or damage), r(t) the rate of repair being done and $\ell(t)$ the load on the system (such as pay-load carried or out-put produced); then ℓ helps S to increase X, whereas r decreases it. Since the forces affecting the system come from random processes, the X-process satisfies a stochastic differential equation. The problem of determining an optimal repair-and-loading policy (r,ℓ) therefore becomes, in principle, a stochastic control problem. But whereas most of the stochastic control literature deals with continuous processes, discontinuous shock processes play a predominant role in our context. Types of results which have been obtained are illustrated with reference to a simple model:

Suppose that the system is subject to damage from two sources; one of these is a simple Poisson process, and the other is a Brownian motion with standard deviation and drift parameter equal to 1. Suppose also that a damage-causing stimulus of unit value produces damage of magnitude

ℓX. Then we have the following simple stochastic differential equation:

$$dX = [\ell X - r] \, dt + \sigma \ell X \, dZ + \ell X \, dY$$

An optimal adaptive stochatic control policy which maximizes the expected value of discounted utility up to a finite horizon H consists of $\ell^* = $ a constant ℓ_0 and $r^* = \alpha(t)X(t)$, where α is an exponential function. In the limit, as σ goes to 0, ℓ_0 also converges to 0, and α to ∞. In other words, if the only process affecting the system is a simple Poisson shock process, then no differentiable policy is admissible. On the other hand, if we admit instantaneous quantal repairs (replacements of components are essentially of this type), then there exists a repair-and- loading policy which prescribes a constant load and guarantees totally risk-free operation upto the horizon H (that is to say, a policy with zero probability of a breakdown). Unfortunately, this policy is independent of H, and is identical with the infinite-horizon policy. Consequently, if the system is facing a short-term emergency, during which it is necessary to place a heavier load on the system than is advisable in long-term usage, this condition is possible only at the cost of a positive probability of total breakdown before the horizon is reached. Further work on problems of this type is in progress.

7. <u>Positive Dependence, Upper Sets, and Multidimensional Partitions</u>.

(Whitaker, A.R. Sampson)

Let U be an upper set contained in the finite discrete lattice L = {1,...,D(1)} X ... X {1, ..D(p)}. Representations for U are obtained and shown to correspond to certain multidimensional partitions of integers. It is shown that for p=3, the number of possible upper sets in

L is $\prod_{i=0}^{D(3)-1} \binom{D(1)+D(2)+i}{D(1)}$. Various other representation and enumeration results are obtained for related settings. Considered are a variety of applications to multivariate positive dependence and various notions in statistics and reliability theory. A paper based on this work will appear in Mathematics of Operations Research.

8. Estimation of Multivariate Distributions Under Stochastic Ordering.

(Whitaker, A.R. Sampson)

Let F and G be the cdf's of two p-dimensional multivariate distributions, such that F is stochastically larger than G. A straightforward derivation is given of the generalized maximum likelihood estimators of F and G based on the random samples from each population. An algorithmic approach to computing these estimators is described and motivating numerical examples are discussed. The special case when F and G correspond to multivariate ordinal contingency tables is also presented. The relationship of these results to those of Robertson and Wright (Ann. Statist. 2 (1974) 528-534) is considered. A paper on this work has been submitted for publication.

9. Computational Aspects of Association for Bivariate Discrete

Distributions. (Whitaker, A.R. Sampson)

For a bivariate discrete probability distribution, P, on the RxC lattice, various aspects for checking association (Esary, Proschan and Walkup (1967)) are considered. A new algorithm is given for verifying whether or not P is associated. The efficiency of this algorithm is obtained and compared to the efficiency of a simple algorithm based on the definition of association When R=C=5 for example, the new algorithm

requires less than 3% of the computations required by the simple algorithm. In obtaining these results, a new set function Q is defined on all upper sets in the lattice; the relationship between P and Q sheds light on the relationship between association and stochastic ordering. Also defined is a computationally important subset of the extreme points of an upper set. A technical report of this work is in preparation.

In addition to the work described above, the investigators have revised a number of grant supported research papers in various stages in the publication process. These include the following:

"Estimating a Survival Curve Based on Nomination Sampling," <u>Journal of the American Statistical Association</u>, 81(1986), 1034-45 (Boyles and Samaniego).

"On Estimating Component Reliability for Systems with Random Redudancy Levels," <u>IEEE-Transactions on Reliability</u>, to appear (Boyles and Samaniego).

"On Estimating Population Characterisitcs From Record-Breaking Observations, II. Nonparametric Results," <u>Naval Research Logistics</u>, to appear (Samaniego and Whitaker).

"On Estimating the Reliability of Systems Subject to Imperfect Repair," <u>Journal of the American Statistical Association</u>, to appear (Whitaker and Samaniego).

"Estimating a Survival Curve when New is Better Than Used in Expectation," submitted for publication (Whitaker and Samaniego).

"New Moment Identities Based on Integrated Survival," submitted for publication (Arnold, Reneau and Samaniego).

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